Fourier Series:

Periodic signal

$$f(t) = f(t + nT_0) \quad n = \pm 1, \pm 2 \ldots \alpha$$

A periodic fn can be expressed as sum of linear independent functions and the sum is also periodic

$$f(t) = a_0 + \sum_{n=1}^{\alpha} \cos(n w_0 t + \theta_n)$$

$$w_0 = 2\pi / T_0 \qquad a_0 = \text{average value}$$

cos ($w_0 n t$) can be expressed as sum of exponents given by Euler identity so,

$$f(t) = a_0 + \sum_{\substack{n=\alpha \\ n \neq 0}}^{\alpha} C_n e^{j w_0 n t} = \sum_{n=-\alpha}^{\alpha} C_n e^{j n w_0 t}$$

n = any integer

$$f(t) = a_0 + \sum_{n=1}^{\alpha} Re\left[(D_n \angle \theta_n) e^{j w_0 n t}\right]$$

$$= a_0 + \sum_{n=1}^{\alpha} Re\left[\left(2C_n e^{j n w_0 t}\right)\right]$$

$$= a_0 + \sum_{n=1}^{\alpha} Re\left[(a_n - j b_n) e^{j n w_0 t}\right]$$

$$\boxed{f(t) = a_0 + \sum_{n=1}^{\alpha} \left(a_n \cos n w_0 t + b_n \sin n w_0 t\right)}$$

$$D_n \angle \theta_n = 2C_n = a_n - j b_n$$

So non-sinusoidal is a complex sum of sinusoids.

The exponential Fourier series

$$f(t) = \sum_{n=\alpha}^{\alpha} c_n e^{jnw_0 t} \quad \text{—— (1)}$$

C_n = Complex phasor = $D_n \angle \theta_n$

integrating both sides of (1) over 1 time period T_0 and multiply by $e^{-jkw_0 t}$

$$\int_{t_0}^{t_0+T_0} f(t) e^{-jkw_0 t} \, dt = \int_{t_0}^{t_0+T_0} \left(\sum_{n=-\alpha}^{\alpha} c_n e^{jw_0 nt} \right) e^{-jkw_0 t}$$

$$= C_k T_0$$

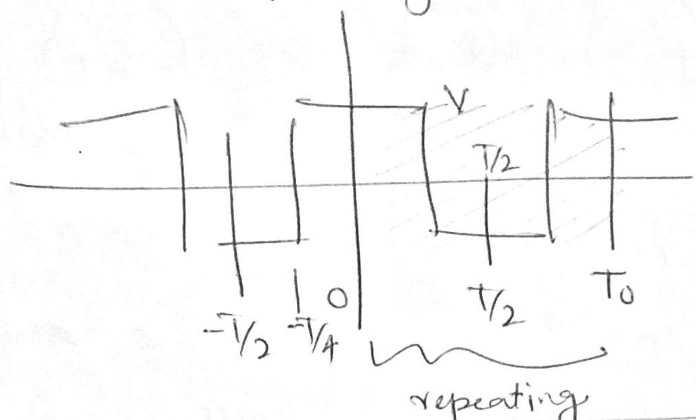
also

$$\int_{t_0}^{T_0+t_0} e^{j(n-k) w_0 t} \, dt = \begin{cases} 0 & n \neq k \\ T_0 & n = k \end{cases}$$

so Fourier coefficients are

$$\boxed{C_n = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} f(t) e^{-jw_0 nt} \, dt}$$

ding Fourier of Fig



$$\boxed{C_n = \frac{1}{T}\int_{-T/2}^{+T/2} f(t)\, e^{-jn\omega_0 t}\, dt}$$

$$= \frac{1}{T}\left[\int_{-T/2}^{-\frac{T}{4}} -V e^{-jn\omega_0 t} dt + \int_{-T/4}^{+T/4} V e^{-jn\omega_0 t} dt + \int_{T/4}^{T/2} (-V) e^{-jn\omega_0 t} dt\right]$$

$$\text{Compart } 1 = \frac{1}{T}\int_{-T/2}^{-T/4} (-V) e^{-jn\omega_1 t}\, dt$$

$$\text{Compart } 2 = \frac{1}{T}\int_{-\frac{T}{4}}^{+T/4} V e^{-jn\omega_0 t}\, dt$$

$$\text{Compa } 3 = \frac{1}{T}\int_{T/4}^{T/2} (-V) e^{-jn\omega_0 t}\, dt$$

$$C_n = \frac{2V}{n\pi T}\left[4 \sin\frac{n\pi}{2} - 2\sin(\pi n)\right]$$

$$C_n = \frac{2V}{n\pi}\sin\left(\frac{n\pi}{2}\right) \qquad \text{odd} = n$$

$$C_n = 0 \qquad \text{even} = n$$

$$C_0 = \frac{1}{T}\int_{-T/2}^{T/2} v(t)\,dt$$ (average value).

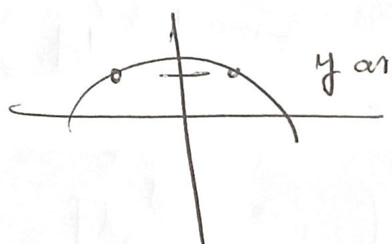
$$C_0 = 0$$

So

$$v(t) = \sum_{\substack{n=1 \\ n \in \text{Odd}}}^{\infty} \frac{4V}{n\pi} \sin\frac{n\pi}{2} \cos n\omega_0 t$$

Properties:

even symmetry $f(t) = f(-t)$



y are equal.

$$a_0 = \frac{2}{T_0}\int_{0}^{T/2} f(t)\,dt$$

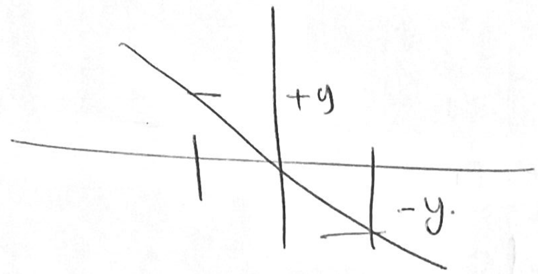
$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{0} f(t)\cos n\omega_0 t\,dt$$

$$b_n = 0$$

odd Symmetry

(3)

$$f(t) = -f(-t)$$



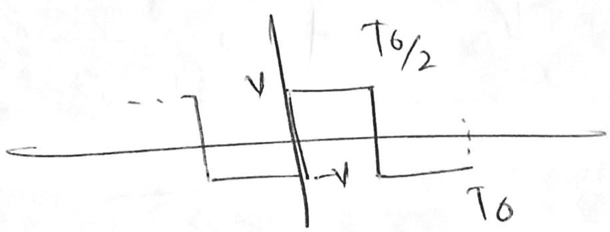
$a_0 = 0$

$a_n = 0$

$$b_n = \frac{4}{T_0}\int_0^{T/2} f(t)\sin n\omega_0 t\,dt$$

Half wave symmetry

$$f(t) = -f\left(t - \frac{T_0}{2}\right)$$



+ve at $(0, T_0/2)$ -ve at $(T_0/2, T_0)$

$a_0 = 0$ (must possess 0 (zero) DC value)

$a_n = b_n = 0$ n: even

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\cos n\omega_0 t\,dt \quad n = odd$$

$$b_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\sin n\omega_0 t\,dt$$

(4)

Fourier Series are used to generate waves for analysis.

- Frequency Spectrum
  - Bands.
  - HP, LP, BPF
- Average Power
- Power dissipation due to component frequencies-

$$P = \frac{1}{T}\int_0^{T_0} v(t)\, i(t)\, dt$$

$$P_{DC} = V_{DC} I_{DC} + \sum_{n=1}^{\infty} \frac{V_n I_n}{2} \cos(\theta_{vn} - \theta_{in})$$
