Two port Network

→ Regarded as black box
→ one input / one output port
→ $I_{in} = -I_{out}$ but no always.
→ Transmission lines, small signal models, T/F filters are example.
- Described by 2x2 matrix
- Z, Y, G and ABCD parameter are used
transmission
- Super position, closed and open tests are used to solve for variables.
→ Energy, power, frequency parameter are linked by ports.
- Reciprocal network : having all passive elements
- Symmetrical network : in and out are dual of one another

impedance

Parameter	we express	In terms of	Matrix
✓ Z	V_1, V_2	I_1, I_2	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$
✓ $Y = \frac{1}{Z}$	I_1, I_2	V_1, V_2	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$
✓ T	V_1, I_1	$V_2, -I_2$	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$
h	V_1, I_2	I_1, V_2	$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$
✓ g	$I_1 V_2$	V_1, I_2	$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$

$Z \to V_1, V_2 \longrightarrow I_1, I_2$

$Y \to I_1, I_2 \longrightarrow V_1, V_2$

$T \to V_1, I_1 \longrightarrow V_2, -I_2$

$G \to I_1, V_2 \longrightarrow V_1, I_2$

} To memem

Impedance Parameter

2)

$$[V] = [Z][I]$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$I_1 = 0$

$V_1 = Z_{12} I_2 \Rightarrow \boxed{Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}}$

$I_1 = 0$

$V_2 = Z_{22} I_2 \Rightarrow \boxed{Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}}$

$I_2 = 0$

$V_1 = Z_{11} I_1$

$\boxed{Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}}$

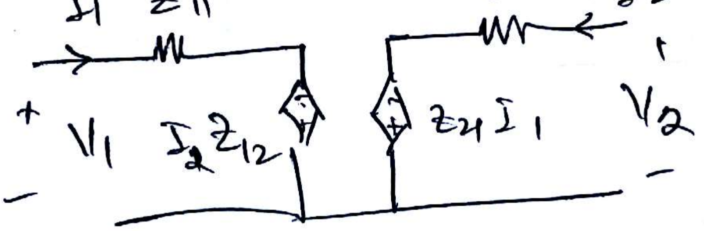
also $\boxed{Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}}$

Z_{12} = reverse transfer function (in 1 due to 2)

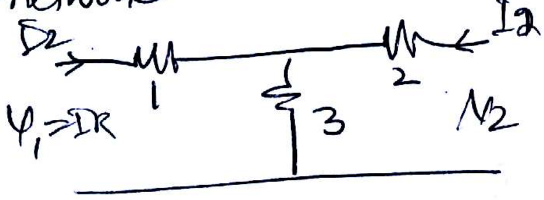
Z_{22} = output driving function

Z_{11} = input driving function

Z_{21} = forward T/F (in 2 due to 1).



Example T-network



Find Z parameter

Solution $V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

KVL long method

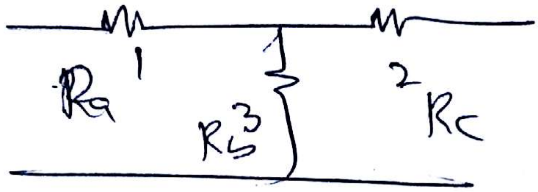
$V_1 = I_1(1+3) + I_2(3)$

$= 4I_1 + 3I_2$

$V_2 = I_1(3) + I_2(2+3)$

$= 3I_1 + 5I_2$

$$[Z] = \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix}$$ ✓



$z_{11} = R_a + R_b$.

$z_{12} = z_{21} = (\text{T port element}) = R_b$. } Method 2

$z_{22} = R_b + R_c$

Solving for Y-parameter

$V_1 = 4I_1 + 3I_2$ —— (1)

$V_2 = 3I_1 + 5I_2$ —— (2)

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$ —— (3)

put $V_2 = 0$ in (2)

$0 = 3I_1 + 5I_2$ (3a)

$I_2 = \frac{-3}{5} I_1$ put in (1)

$V_1 = 4I_1 + 3\left(\frac{-3}{5}\right)I_1$

$V_1 = 4I_1 - \frac{9}{5} I_1$

$V_1 = \frac{20I_1 - 9I_1}{5}$

$\frac{V_1}{I_1} = +\frac{11}{5}$

$\frac{I_1}{V_1} = Y_{11} = \frac{5}{11}$

$Y_{12} = \frac{I_1}{V_2}$

use equation (2) and (3a)

$V_2 = 3I_1 + 5\left(\frac{-3}{5}\right)I_1$ Check?

$\frac{V_2}{I_1} = 3 - 3 = 0$ $\quad Y_{12} = \frac{I_1}{V_2} = \infty$

(3)

Y- parameters·

a) $V = I\,Z$ actually.

$$[V] = [I][Z]$$

$$[Y][V] = I$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$I_1 = V_1 Y_{11} + V_2 Y_{12}$$

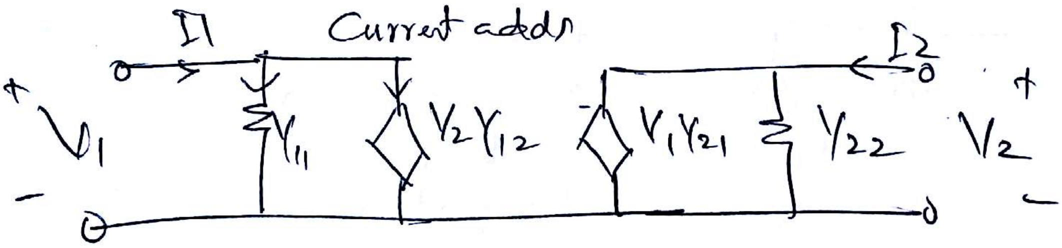
$$I_2 = V_1 Y_{21} + V_2 Y_{22}$$

put $V_1 = 0$

$$Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0}, \quad Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$

put $V_2 = 0$

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0}, \quad Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$



$$[Y] = [Z]^{-1}$$

another

not o/c

Y not zero

H-parameter

$$\begin{matrix}V_1\\I_2\end{matrix} = \begin{bmatrix}h_{11} & h_{12}\\h_{21} & h_{22}\end{bmatrix}\begin{bmatrix}I_1\\V_2\end{bmatrix}$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$I_1 = 0$

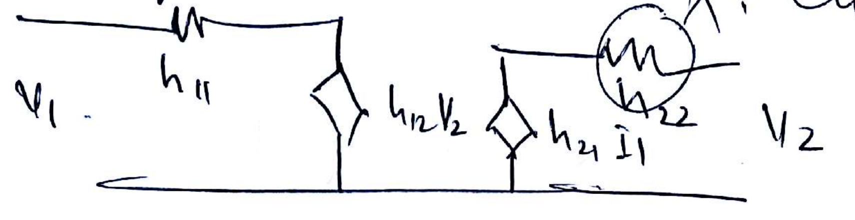
$$\frac{V_1}{V_2} = h_{12}, \quad h_{21} = \frac{I_2}{I_1}$$

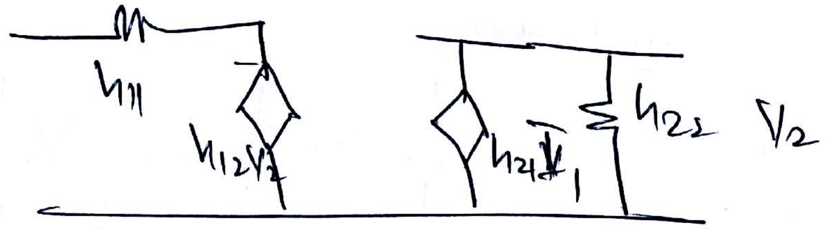
$V_2 = 0$

$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} \qquad h_{22} = \frac{I_2}{V_2}$$

Voltage

X. Currents





④

T-parameter.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$V_2 = 0$

$B = -\frac{V_1}{I_2}\Big|_{V_2=0}$ $D = -\frac{I_1}{I_2}\Big|_{V_2=0}$

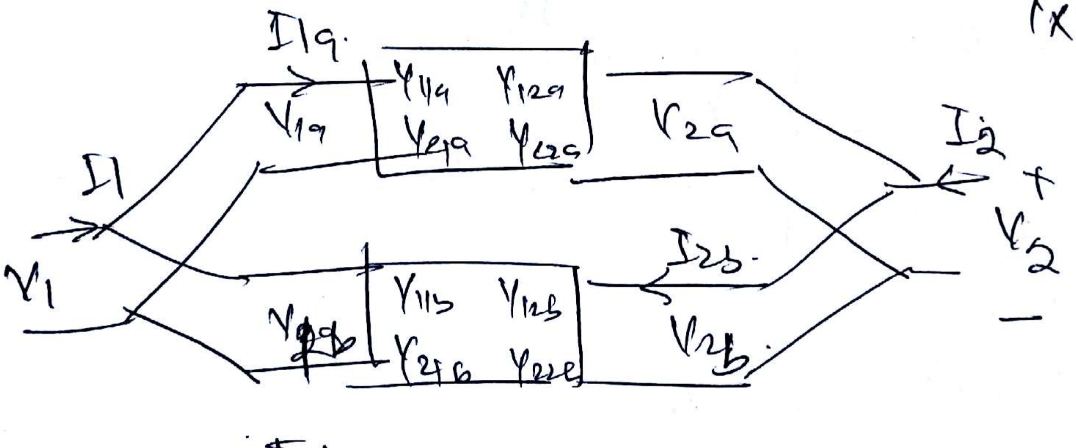
$I_2 = 0$

$A = \frac{V_1}{V_2}\Big|_{I_2=0}$ $C = \frac{I_1}{V_2}\Big|_{I_2=0}$

series connect

parallel con

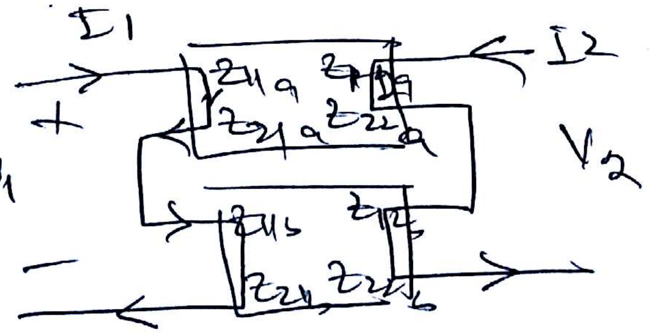
Cascade connt



add

$Y_{xy} = Y_{xx} + Y_{yy}$

parallel

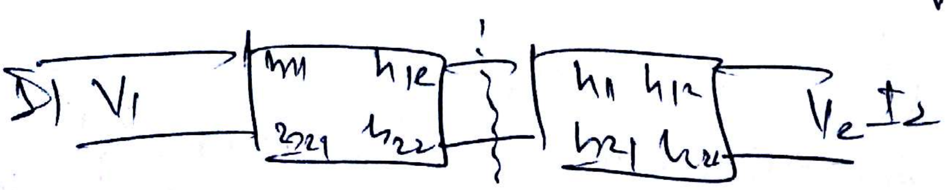


add

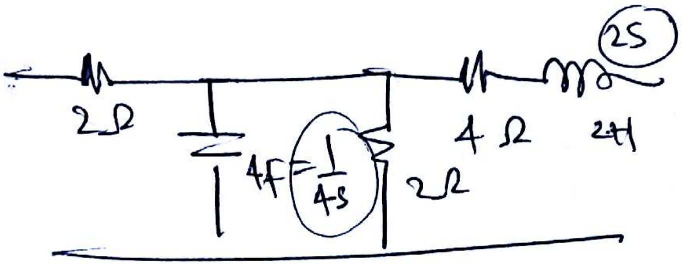
$Z_{xy} = Z_{xx} + Z_{yy}$

multip

$h_{xy} = [h_{xx}][h_{yy}]$

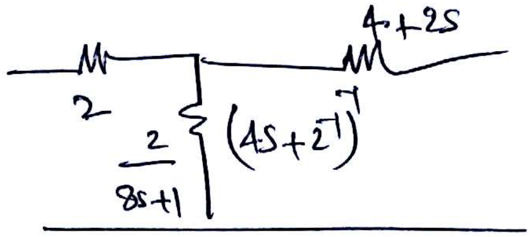


Example



Find $[Z]$

becomes



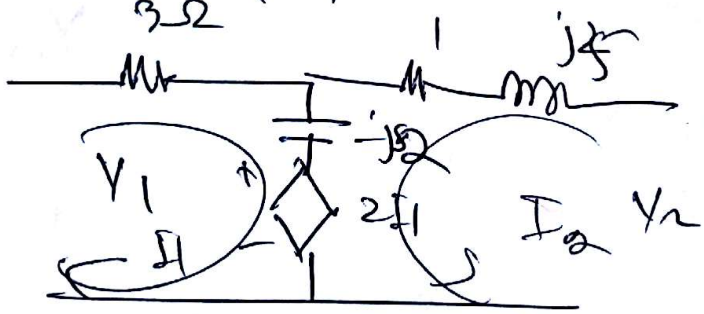
$$z_{11} = 2 + \frac{2}{8s+1} \qquad z_{12} = \frac{2}{8s+1} \qquad z_{21} = \frac{2}{8s+1}$$

$$z_{22} = 4+2s+\frac{2}{8s+1}$$

$$[Z] = \begin{bmatrix} 2+\frac{2}{8s+1} & 2/8s+1 \\ 2/8s+1 & 4+2s+\frac{2}{8s+1} \end{bmatrix}$$

$$[Y] = \frac{1}{[Z]} = [Z]^{-1}$$

Example



$$V_1 = 3I_1 - j2(I_1+I_2) + 2I_1$$

$$V_1 = (5-j2)I_1 - (j2)I_2.$$

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$z_{11} = 5-2j.$$

$$z_{12} = 2j$$

$$z_{21} = 2j$$

$$z_{22} = ?$$

solve for second loop

$$V_2 = I_1(-2j) + 2I_1 + I_2(5j+1-2j)$$

Scanned by CamScanner

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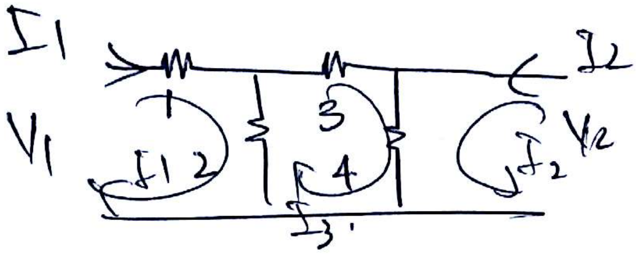

Z: loop
Y = Node.

Loop 1

$V_1 = I_1(1+2) - I_3(2)$

$V_1 = 3I_1 - 2I_3$ —— (1)

$0 = -I_1(2) + I_2(4) + I_3(2+3+4)$

$0 = -2I_1 + 4I_2 + 9I_3$ —— (2)

$V_2 = 4I_2 + 4I_3$

$4I_2 = V_2 - 4I_3$ —— (3) put in (2) / (1)

$I_2 = \frac{V_2 - 4I_3}{4}$ eleminates I_2

$I_3 = \frac{V_2 - 4I_2}{4}$ elemnts I_3 ✓

$V_1 = 3I_1 - 2\left(\frac{V_2 - 4I_2}{4}\right)$

$V_1 = 3I_1 - \frac{2V_2}{4} - \frac{2 \cdot 4I_2}{4}$

$V_1 = 3I_1 - \frac{V_2}{2} - 4I_2$

$2V_1 = 6I_1 - V_2 - 8I_2$

$\boxed{6I_1 + 4I_2 = 2V_1 + V_2}$ similarly put in (3)

~~for $I_1 = 0$~~

~~$4I_2 = 2V_1 + V_2$~~

from both (1) and (3) $23I_1 + 8I_2 = 9V_1$ (B)

$I_1 = 0$

$\frac{V_1}{I_2} = Z_{12} = \frac{8}{9}$ Continue for rest