Ex B.4 $te^{-(t-1)}u(t-1) - e^{-(t-1)}u(t-1)$

$e^{-(t-1)}[t-1]u(t-1)$

since: $\mathcal{L}(e^{-at}) = \frac{1}{s+a}$ Real distinct 1

$\mathcal{L}(t) = \frac{1}{s^2}$ Real repealed 2

$\mathcal{L}(f(t-t_0)) = e^{-t_0 s}F(s)$ Complex conjugate 3

so $t_0 = 1$

$$\boxed{F(s) = \frac{1}{(s+1)^2}e^{-1s}}$$

B.5 $f(t) = e^{-4t}(t - e^{-t})$

$= te^{-4t} - e^{-5t}$

$= \frac{1}{(s+4)^2} - \frac{1}{s+5}$

$\mathcal{L}(t) = \frac{1}{s^2}$.

$\mathcal{L}(e^{-4t}) = \frac{1}{s+4}$

EX B.7 $F(s) = 12(s+2)/s(s+1)$

$$\frac{12(s+2)}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$A = \frac{12(s+2)}{s(s+1)} * s \Big|_{s=0} = \frac{12(s+2)}{s+1} = \frac{12(2)}{1} = 12.2 = 24$$

$$B = (s+1)\frac{12(s+2)}{s(s+1)}\Bigg|_{s=-1}$$

$$B = \frac{12(-1+2)}{-1} = \frac{12}{-1} = -12$$

$$f(t) = \frac{24}{s+0} + \frac{-12}{s+1}$$

$$\mathcal{L}(f(t)) = 24e^{-0t} + (-12)(e^{-1t})$$

$$\mathcal{L}(f(t)) = [24 - 12e^{-t}]\, u(t) \leftarrow \text{for real world signal } \int_0^{\infty}$$

E13.9 $F(s) = \dfrac{s}{s^2+4s+8}$

$s^2+4s+8 = (s+2+2j)(s+2-2j)$; using Calculator

$$F(s) = \frac{s}{(s+2+2j)(s+2-2j)}$$

$$\frac{s}{(s+2+2j)(s+2-2j)} = \frac{A}{s+2+2j} + \frac{B}{s+2-2j}$$

$$A = \frac{s}{s+2-2j}\Bigg|_{s=-2-2j} = \frac{-2-2j}{\cancel{-2}-2j+\cancel{2}-2j} = -\left(\frac{2+2j}{4j}\right)$$

$$= \frac{1}{2} - \frac{1}{2}j$$

$$= \frac{\sqrt{2}}{2}\angle 45^0$$

$$F(s) = \frac{\sqrt{2}\,e^{j45}e^{-2t} + e^{-j45^0}e^{+2t}}{2}$$

$$= 1.41\, e^{-2t}(\cos(2t+45^0))\, u(t)$$

$A(s+ \ldots$

$(\ldots$

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$$\frac{4(s+3)}{(s+1)(s^2+2s+5)}$$

$$s^2+2s+5 = (s+1+2j)(s+1-2j)$$

$$\frac{4(s+3)}{(s+1)(s^2+2s+5)} = \frac{A}{s+1} + \frac{B}{(s+1+2j)} + \frac{C}{(s+1-2j)}$$

$$A e^{-1t} + e^{j\theta}\left[\frac{Be^{jwt} + B^{*}e^{-jwt}}{2}\right]$$

$$f(t) = 2e^{-t} + 2\sqrt{2}\, e^{-t}\cos(t-135^{0})]\,u(t).$$

B.11

$$F(s) = \frac{s}{(s+1)^2}$$

$$\frac{s}{s+1} = \frac{A}{s+1} + \frac{B}{(s+1)^2}$$

$$B = \frac{s}{(s+1)^2} * (s+1)^2 \Big|_{s=-1}$$

$$B = s\Big|_{s=-1} = -1$$

$$A = \frac{d}{dt}(s+1)^2 \cdot \frac{s}{(s+1)^2} = 1 \Big|_{s=-1} = 1$$

$$\frac{s}{s+1} = \frac{1}{s+1} - \frac{1}{(s+1)^2}$$

$$= (e^{-1t} - te^{-1t})\,u(t).$$

13.12

$$F(s) = \frac{s+2}{s^2(s+1)}$$

$f(t) = ?$

$$F(s) = \frac{s+2}{s^2(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1}$$

$$C = \frac{s+2}{s^2(s+1)} * (s+1) \Big|_{s=-1} = \frac{-1+2}{(-1)^2} = \frac{1}{1} = 1$$

$$B = \frac{s+2}{\cancel{s^2}(s+1)} * \cancel{s^2} \Big|_{s=0} = \frac{2}{1} = 2$$

$$A = \frac{d}{ds}\left(\frac{s+2}{s+1}\right)\Big|_{s=0} = \frac{(s+1)(1) - (s+2)(1)}{(s+1)^2}\Big|_{s=0}$$

$$A = \frac{(0+1)(1) - (2)(1)}{(0+1)^2} = \frac{1-2}{1} = -1$$

$$\mathcal{L}^{-1}\left(\frac{s+2}{s^2(s+1)}\right) = \mathcal{L}^{-1}\left(\frac{-1}{s+0} + \frac{2}{(s+0)^2} + \frac{1}{s+1}\right)$$

$$f(t) = \left(-e^{-0t} + 2te^{-0t} + e^{-t}\right) u(t)$$

$$f(t) = \left(1 - 2t + e^{-t}\right)(u(t))$$

Q.13

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$$\frac{100}{s^3(s+5)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s+5}$$

$A = 0.8$, $B = -4$, $C = 20$, $D = 0.8$

$$\mathcal{L}(F(t)) = \mathcal{L}\left(\frac{0.8}{s} + \frac{-4}{s^2} + \frac{20}{s^3} + \frac{0.8}{s+5}\right)$$

$$f(t) = 0.8e^{-0t} - 4e^{0}t + 10t^2 + 0.8e^{-5t}$$

$$f(t) = \left(0.8 - 4t + 10t^2 + 0.8e^{-5t}\right)u(t).$$

Initial value Theorem and Final value Theorem

We can find initial value of a function by putting $t=0$ and the final value by putting $t \to \infty$

- A signal that covers all frequency domain is an impulse on time domain
- A signal that covers all time domain is an impulse in frequency domain.

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

13.13 $F(s) = \dfrac{10(s+1)}{s(s^2+2s+2)}$

to find initial and final value of function

$f(0) = \lim_{s\to\infty} sF(s)$

t=0 initial value

$= \dfrac{10(s+1)}{s^2+2s+2}\Big|_{s\to\infty}$

$f(0) = \dfrac{X}{s^2} = \dfrac{X}{\infty} = 0$

$f(\infty) = \lim_{s\to 0} sF(s)$

$= \dfrac{10(s+1)}{s^2+2s+2} = \dfrac{10}{2}$

$f(\infty) = 5$

$s^2+2s+2s+4$

$s(s+2)+2(s+2)$

$(s+2)^2$

E13.16 $y(t) = ?$

$\dfrac{dy}{dt} + 4y(t) + 4\int_0^t y(x)\,dx = 10u(t)$ $y(0) = 10$

$sY(s) - y(0) + 4Y(s) + \dfrac{4}{s}Y(s) + \dfrac{y(0)}{s} = 10\left(\dfrac{1}{s}\right)$

$Y(s)\left(s^2 + 4s + 4 + \dfrac{10}{s}\right) = \dfrac{10}{s}$

$Y(s) = \dfrac{1}{s^2+4s+4}$

$Y(s) = \dfrac{10}{s+4+\dfrac{4}{s} + \dfrac{10}{s}}$ neglecting for simplification

$(Bte^{-2t} + Ae^{-2t})u(t)$

$\dfrac{A}{s+2} + \dfrac{B}{(s+2)^2}$

$= \dfrac{10s}{s^2+4s+4} = \dfrac{10s}{s^2+2s+2s+4} = \dfrac{10s}{s(s+2)+2(s+2)}$

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$$\frac{10s}{s^2+4s+4} = \frac{A}{s+2} + \frac{B}{(s+2)^2}$$

$$A = \frac{d}{dt}(s+2)^2\frac{10s}{(s+2)^2} = \frac{d}{dt}(10s) = 10$$

$$B = (s+2)^2\frac{10s}{(s+2)^2} = 10s\Big|_{s=-2} = -20$$

$$\mathcal{L}^{-1}\left(\frac{10s}{s^2+4s+4}\right) = \left(10e^{-2t} - 20e^{-2t}\right)u(t)$$

Due to neglecting $\frac{10}{s}$

$\boxed{\text{Check } [B] \text{ p12}}$

Remember $\mathcal{L}\left(\int_x^0 y(x)\,dx\right) = \frac{Y(s)}{s} + \frac{y(0)}{s}$

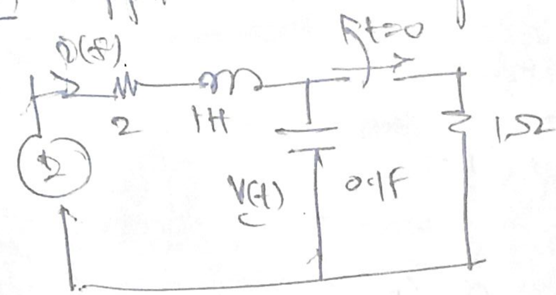
$$\mathcal{L}\left(\frac{d}{dt}y(t)\right) = sY(s) - y(0)$$

→ Integrals add up residue $f(t)$ and always acts at $t=0$ and continues till $t\to\infty$ so integration involves $\frac{1}{s}$ from $u(t)$

→ Differentiation takes difference of initial values/residue.

$\lim_{t\to 0} f(t)$ is called residue.

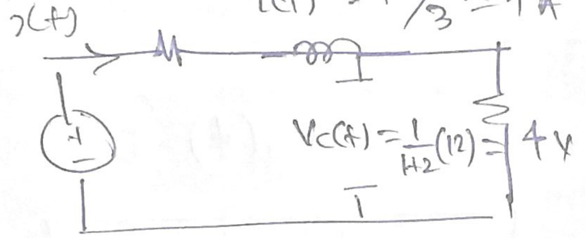
13.14 Application example



find $i(t)$ for $t > 0$

at $t \leq 0$

$i(t) = 12/3 = 4A$

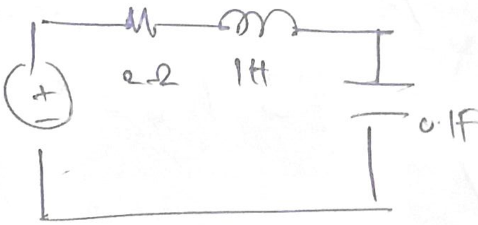


$i(t) = 4A$, $V_c(t) = 4$ volts.

for $t > 0$

inductor has residual $I(0)$

capacitor has residual voltage $\frac{V(0)}{S}$



$$12u(t) = i(t)2 + 1\frac{di(t)}{dt} + \frac{1}{0.1}\int_0^t i(x)dx$$

$$\frac{12}{S} = I(S)2 + 1.SI(S) - i(0) + 10\underbrace{\left[\frac{I(S)}{S}\right]}_{\text{Voltage}} + \frac{V(0)}{S}$$

$$\frac{12}{S} = I(S)\left(2 + S + \frac{10}{S}\right) - 4 + \frac{4}{S}$$

$$I(S) = \frac{4(S+2)}{S^2+2S+10} = \frac{4(S+2)}{(S+1-3j)(S+1+3j)}$$

$K_1 = 2.11\angle{-18.5}$

$i(t) = 2(2.11)e^{-t}\cos(3t - 18.4^\circ)u(t)$ A