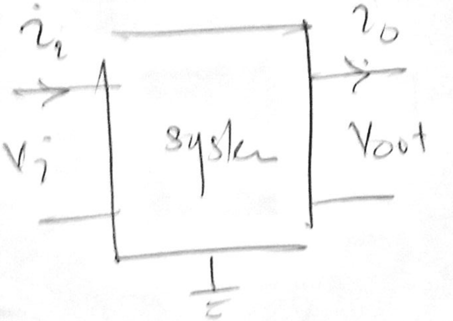
Transfer function

The characteristics of the system are given by a mathematical function that includes C, L and R.
This function is dependent upon frequency.

- T/F is also called Network function / System function



$\frac{V_o(s)}{V_{in}(s)} = G_v(s)$ Gain

$\frac{V_o(s)}{I_{in}(s)} = Y_t(s)$ Trans ad

$\frac{I_{out}(s)}{V_{in}(s)} = Y(s)$ Trans imped

$\frac{I_o(s)}{I_{in}(s)} = G_i(s)$ Gain

Response = Transfer function * input signal

- Semi logarithmic scale is used for finding frequency response on a plot

$$dB = 10\log\frac{P_2}{P_1} = 20\log\frac{V_2}{V_1} = 20\log\left(\frac{I_2}{I_1}\right)$$

$$H(j\omega) = \frac{\text{poles}}{\text{zeros}} * \overset{G}{\text{gain}}$$

$$= \frac{K_0(a_0s^0 + a_1s^1 + a_2s^2 + \ldots)}{b_0s^0 + b_1s^1 + b_2s^2 + \ldots}$$

K0 is independent of frequency

simple pole: $\left(\frac{1}{T} + s\right)$ quadratic pole: $(1 + 2\overset{\substack{\text{zeta}\\ \downarrow}}{\zeta}\omega T + (j\omega T)^2)$

What is Break frequency? (Assignment).

Quality Factor

Reciprocal of power factor

$\cos\theta = \frac{R}{Z}$

$\frac{1}{\cos\theta} = Q = \frac{Z}{R}$

$$\boxed{Q = \frac{\text{Energy stored}}{\text{Energy dissipated}}}$$

$Q_L = \frac{R + jwL}{R} = \left|\frac{jwL}{R}\right| \quad R \ll wL$

$Q = \frac{wL}{R}$

$Q_C = \left|\frac{R + 1/jwC}{R}\right| = \frac{1}{wCR} \quad R \ll wC$

$Q_C = Q_L$

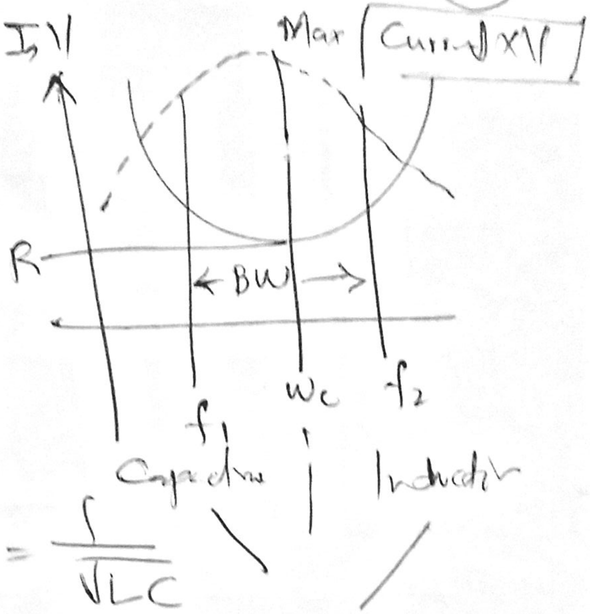
$Q^2 = Q_L \cdot Q_C = \frac{\cancel{w}L}{\cancel{w}CR \cdot R} = \frac{L}{R^2C}$

$Q = \sqrt{\frac{L}{R^2C}} = \frac{1}{R}\sqrt{\frac{L}{C}}$

Also $Q = \frac{f_{central}}{f_2 - f_1} = \frac{f_c}{\Delta f} = \frac{f_c}{\text{Bandwidth}}$

Series Resonace Circuit

Resonance frequency



when $|X_L| = |X_C|$ R

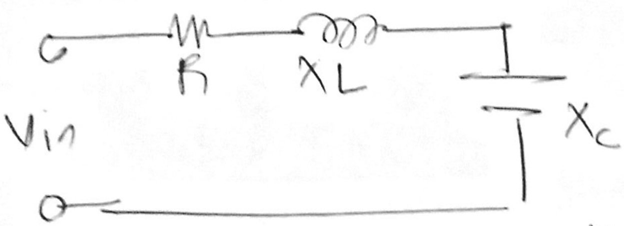
$$w_c L = \frac{1}{w C}$$

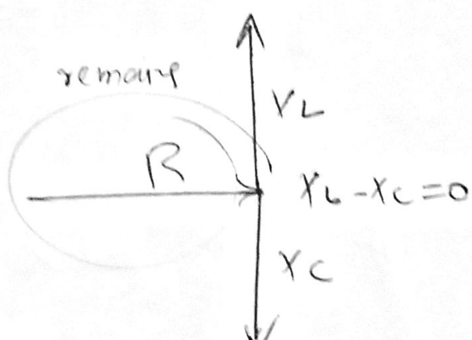
$$w^2 = \frac{1}{LC}$$

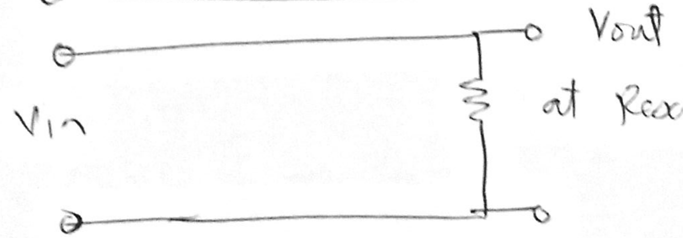
$$w_c = \sqrt{\frac{1}{LC}} = \frac{1}{\sqrt{LC}}$$

$$2\pi f = \frac{1}{\sqrt{LC}}$$

$$f_c = \frac{1}{2\pi\sqrt{LC}}$$

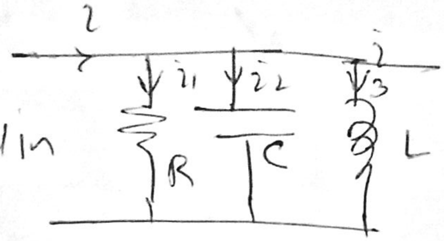






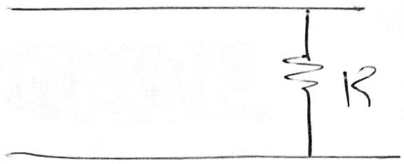
at Resonance frequency

w	$X_L(wL)$	$X_C(\frac{1}{wC})$	R	Z	V	I
0	0	∞	R	∞	—	0
1	L	$\frac{1}{C}$	R	$R + L + \frac{1}{C}$	$\frac{V_1}{I_1}$	$I_1 = \frac{V_{in}}{R + \frac{1}{C} + R}$
10	10L	$\frac{1}{10C}$	R	$10L + R + \frac{1}{10L}$	$\frac{V_{10}}{I_{10}}$	$I_{10} = \frac{V_{in}}{R + Z_{cL}}$
∞	∞	$\frac{1}{\infty} = 0$	R	∞	—	0
$\Rightarrow W_c$	jW_cL	$-j\frac{1}{W_c}$	R	R	$\frac{V}{I}$	$I_{max} = \frac{V_{in}}{R}$



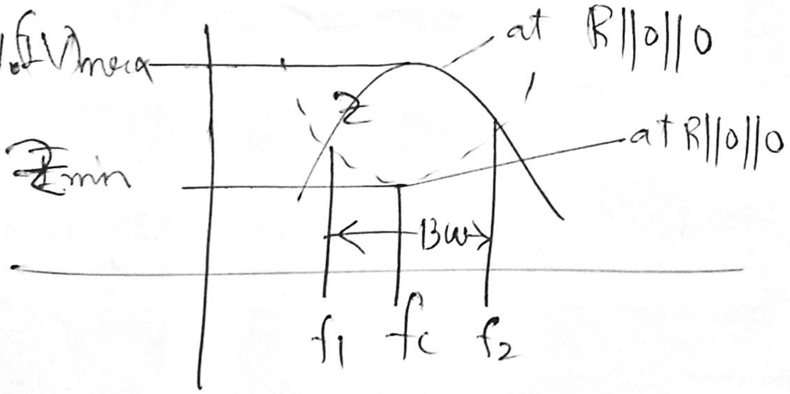
i has multiple paths to flow form Z is low

at $X_L = X_C$



i has only one path for its flow

Z = R is high



BW: Bandwidth

$\boxed{\text{no parallel path results high Z}}$

Series (reference I)

at ω_c

$X_C = X_L$

$Z = R$ = minimum

Current is in phase with voltage so power transfer is maximum

$\omega < \omega_c$

Voltage lags current

$\omega > \omega_c$

Voltage leads current

Parallel (reference V)

at ω_c

$X_C = X_L$

$Z = R$ = maximum

$P = IV$ I in phase with V

so max power transfer

$\omega < \omega_c$

I lags V

$\omega > \omega_c$

I leads V.